ON NON-GAUSSIAN NOISE IN ANGLES-ONLY NAVIGATION

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Abstract. *Angles-only navigation has a broad utility within spaceflight due to the low cost and high reliability of cameras. These cameras provide imagery to targets of interest (such as a planet, surface features/craters, another spacecraft, etc.) so that a navigation system can estimate the camera's state with respect to that target. However, the image processing needed to produce the necessary bearing angles often induces non-Gaussian estimation errors. This work discusses the use of two newly developed filters that efficiently provide (approximately) Bayes' rule optimal conditional mean and covariance estimates using measurements corrupted by uniform or heavy-tailed (Cauchy) noise.*

Background. Due to their low size, weight, and power requirements and relatively low financial cost, cameras have stood out as attractive navigation sensors for a long time. As such, cameras have received significant attention for use in (i) terrain relative navigation, (ii) rendezvous, proximity operations, and docking (RPOD), and (iii) ground-based satellite tracking, just to name a few. These cameras have different driving technologies and formats, such as the difference between optical, thermal, and event cameras, but (in the context of the present discussion) their navigation product is the same: bearing angles to reference points. Since an image is two-dimensional, it is the task of navigation to resolve the indirect relationship between these bearing angles and the three-dimensional space to which the camera, and the imaged feature(s), belong. When only these bearing angles (and, typically, an inertial measurement unit) are available, this is referred to as "angles-only navigation", named so to remind one that there is an inherent ambiguity along the range direction of each observation (the camera boresight).

Remark 1 *There are techniques that attempt to match the feature of interest to some a priori known model, driven by techniques such as normalized cross-correlation/template matching or machine learning. The present discussion restricts focus to methods that utilize bearing angles in the image plane for navigation.*

Navigation algorithms that can directly process the images collected by a camera are exceedingly uncommon, mostly due to the sheer amount of data contained by even a single image. A relatively modest 2,000-by-1,000 pixel image itself contains 2,000,000 pixels! That is a lot of information for an estimation scheme to try to ingest directly. As such, the image must be subjected to pre-processing that converts the content of the image to a collection of reference bearings for use within a navigation algorithm. Note that the nature of this pre-processing varies widely by application. For star field imagery, it is common to binarize the image to compute connected components of each distinct, desired feature in the image. For crater-based navigation, specialized crater-finding algorithms are deployed to search for unique craters in the image. Things such as stars tend to have a structure within an image which is that of a blooming circle, meaning that its structure is generally well-approximated as a Gaussian. Other things, such as artificial space objects like satellites or natural surface features like craters, often contain structure that induces non-Gaussian measurement errors.

Non-Gaussian measurement errors have long been known to degrade traditional estimation techniques based upon the Kalman filter (KF). For a linear system, when all state and noise densities are Gaussian, the Kalman filter *exactly* represents the *true* posterior, whereas when any state or noise density is non-Gaussian it can only be claimed that the first two central moments are well-represented. Of course, in the real-world, it is difficult to make any concrete claims about Gaussianity in the underlying system noises, but it suffices to say that the presence of any non-Gaussian effects degrade the KF's estimation performance. Additionally, non-Gaussian measurement noises are well-known to cause algorithmic stability issues, thus supporting the ubiquity of residual editing in KF applications.[1]

Methodology. This work seeks to address some of the challenges in camera-based navigation induced by non-Gaussian noise by using two newly-developed filters for non-Gaussian noise:

1. measurements corrupted with uniform noise[2]
2. measurements corrupted with heavy-tailed noise (in particular, Cauchy)[3]

Both of these filters have been demonstrated to produce approximately Bayes' rule optimal estimation performance with similar runtimes as the KF, whereas existing techniques based upon particle filtering[4] or Gaussian sums[5] provide similarly optimal performance but (usually) at a dramatically increased runtime cost. Furthermore, the new filters (derived as approximations of the conditional mean and covariance of the true Bayesian posterior) are structurally identical to the KF in that they are "mean and covariance in, mean and covariance out." In this sense, they are attractive plug-and-play options to test in existing KF-based systems.

The Gaussian, uniform, and Cauchy distributions are widely known, but an example of the difference in their pdf structure is given in Fig. 1 for discussion. This figure depicts the three densities which have been "matched," where we depict Cauchy ($\gamma = 1$), Gaussian ($\sigma = 1.3898\gamma$ via least-squares fit to the Cauchy), and uniform (on $[-a, a]$ where $a = \sqrt{12}\sigma/2$ via variance match to the Gaussian) distributions. Note that, while being statistically matched, the structure of these pdfs are wildly different and, thus, their frequency content is similarly different. This frequency content provides the behavior of the noise signature, so all three of these densities provide very different noise signatures. Of course, this is all obvious, but the point is made for emphasis: simply inspecting these three pdfs, with the Cauchy in particular, allows one to see the challenges that can be induced with non-Gaussian noise if a Gaussian is assumed. We see that the Cauchy is a particularly attractive noise model for common real-world systems corrupted with heavy-tailed measurement noise due to is extremely heavy tails.

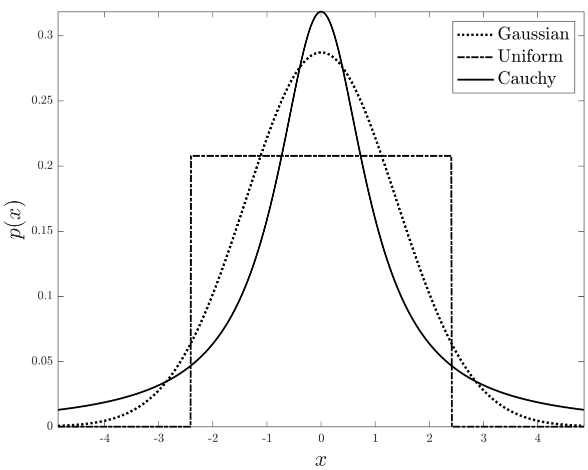

Figure 1. Illustrative example of zero-mean pdfs that have been statistically "matched."

Proposed Approach. This work assesses the performance enhancements offered by the new filters over traditional KF methods for angles-only navigation. These methods apply to any camera-based scenario, but here we consider a representative RPOD example, wherein a chaser spacecraft seeks to estimate its state with respect to a target vehicle. The relative range between the two vehicles is such that the target vehicle is sufficiently large so as to produce an imaged profile that results in non-Gaussian noise in the bearing angles. In addition, this work considers a particularly challenging case that supposes the image pre-processing produces two bearing angles for the single target of interest (such as providing one bearing to the central body of the spacecraft and a second bearing to a solar panel).

The numerical performance of the KF and the new filters are compared in Monte Carlo studies to determine the performance differences between the two. Discussion is presented regarding relevant advantages and drawbacks of one approach versus another. Conclusions are provided to assess forward work.

References.

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