AN H-MATRIX POSE ESTIMATION WITH TRN APPLICATIONS

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Abstract. *We present a camera pose estimation algorithm using the H-matrix with unique compensation for non-planar feature correspondences. A normal pose estimation procedure involves finding correspondences (feature matches) between the image features and their 3d counterparts in the world frame and then estimating the camera pose using the geometry of those correspondences. However, the set of feature correspondences between the image and 3d world often contains outliers which require removal before calculating the final pose estimation. In this paper we offer a simple outlier rejection and pose estimation approach using a convenient property between a virtual plane and the camera pose using the H-matrix formulation. The proposed solution is faster and more robust than other solutions such as the F-matrix[4] or collinearity model[5] when using the RANdom Sample Consensus (RANSAC)[6] technique.*

Introduction. Terrain relative navigation (TRN) has become an important capability for safe and precise spacecraft landing on another planetary body. The onboard TRN system carries a premade terrain map of landing site to which the descent image is matched to estimate the spacecraft pose (both attitude and position). In the normal situation when the spacecraft attitude and altitude are known, feature matching, outlier rejection and pose estimation operations can be greatly simplified [1]. However, when spacecraft state information is absent, the TRN problem complexity increases because the correspondence search is multi-dimensional (position + attitude) rather than a 2D search [1]. In this case, descriptor-based feature matching becomes a viable solution [2]. Multi-dimensional search combined with a large search region introduces more outliers, making quick and reliable outlier rejection critical to algorithm performance. In this paper, we propose a novel outlier rejection and pose estimation algorithm for the TRN problem where the spacecraft state is not available. This paper assumes the correspondences between the image and map have been identified and include false matches (outliers) requiring identification.

H-matrix Introduction. Let the camera is at C(c_x, c_y, c_z) and the rotation from world to the camera frame is *cRw*. Therefore the transformation between a point in 3d world (*P*) to the camera frame can be expressed as

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \cong cRw(P - C) \qquad [1]$$

We start the problem from the simple situation first. Let's a *point* lies on a XY plane, where Z = 0, then $P_0 = (X_1, Y_1, 0)^T$.

Let r_1, r_2, r_3 be the three columns of *cRw* , $C_c = -{}_cR_wC_w$, and $P = (X, Y, 1)^T$ then we have

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \cong cRw(P_0 - C) = (r_1 \quad r_2 \quad C_c)P = HP = (h_1 \quad h_2 \quad h_3) \qquad [2]$$

Clearly the first and second column of *H* are the first two columns of *cRw* and therefore their norm should be 1 and their dot product should be 0 as

$$\|r_1\| = \|h_1\| = 1 \quad \|r_2\| = \|h_2\| = 1 \quad r_1.r_2 = h_1 \cdot h_2 = 0$$

This is a special case of homography between a Z= 0 plane and the image plane and we name it H-Matrix.

For any given H-Matrix, H_e, we can recover its full H-matrix by multiplying a scale factor using this property as

$$H = sH_e \ \ where\ s = 2.0/(\|h_{e1}\| + \|h_{e2}\|)$$

Then it is very easy to derive the camera position *C*(c_x, c_y, c_z) from a full H-Matrix since

$$A = H^TH = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & -c_x \\ 0 & 1 & -c_y \\ -c_x & -c_y & \|C\|^2 \end{pmatrix} \qquad [3]$$

Clearly, Matrix *A* contains camera position only. The conditions of $a_{12} = a_{21} = 0,\ a_{11} = 1, a_{22} = 1\ and\ a_{33} - c_x^2 - c_y^2 > 0$ can be used to validate the H-matrix condition. When a H-Matrix fails these condition check, it indicates that the input data either ill conditioned such as all points are colinear or wrong, where the input contains outliers. In this case, we abandon this H matrix and move to another data set.

When if passes the check, there are two mirrored solution as

$$\left(-a_{13} \quad -a_{23} \quad \pm\sqrt[2]{a_{33} - c_x^2 - c_y^2}\right) \qquad [4]$$

The true solution can be picked under the real application condition, such as the camera must be above the surface of the ground.

Then the rotation can be obtained as

$$cRw = H(I - CN^T)^{-1} \qquad [5]$$

Where *N* = (0, 0, 1);

H-Matrix (*H*) for a general case where 3d points are not coplanar. Let's assume a set of points between an image and world has been found {p_i}➔{P_i}, where lower case *p* is the point on the image and upper case *P* is the 3d point in world frame. We assume that more than 4 points are found (10s to 100s of points).

For a given set of 3d points we can easily find a translation P_m and rotation ${}_lR_w$ to convert those point into a local coordinate frame where Z axis is aligned with the smallest eigenvector of the point clouds.

As $P_i' = lRw(P_i \; - P_m)$

First, let all Z value of $\{P'_i\}$ be 0, so that an H-Matrix (H_0) can be constructed by these points via a least-squares method.

We then can obtain an initial pose from the H_0-Matrix (cRl_0, C_0) using equation 4 to 5.

For any given point in $\{P'_i\}$ P'(X', Y', Z'), we separate it into two components as $P' = P'_0 + P_z$ where P'_0=(X', Y', 0) and P_z = (0, 0, Z'), then we can rewrite the equation as

$$\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \cong cRl(P' - C) = cRl(P_0 - C) + cRlP_z$$
$$= c(r_1 \quad r_2 \quad C_c)P_0 + P_{zc}$$

Where $P_{zc} = {}_cR_wP_z$.

The first part of the equation is the H-Matrix for plane Z = 0 and the second component is the compensation component due to nonplanarity of those 3d points. Because an initial ${}_cR_{w0}$ is obtained already the compensation component can be approximated by $P_{zc} = cRl_0P_z$. Then the updated H-Matrix can be constructed using regular method after adding the compensation. However, the compensation has to be scaled by $s = 1/h_{33}$.

Therefore, we have the following procedural method for pose estimation:

1. For a given set of 3d point find a rotation and translation to convert them into the local frame;
2. Use the x and y components of 3d points (in the local frame) to construct a H-Matrix and compute the initial pose(cRl_0, and Cl_0);
3. Reconstruct new H-Matrix by adding the compensation component P_{zc};
4. Compute the pose using the newly compensated H-Matrix;
5. Compute the reprojection error of 3D points;
6. If the reprojection error is less than tolerance, go to the next step otherwise iterate back to step 3;
7. Convert the pose from local frame to world frame.

Normally, this convergence is very fast and it takes few iterations to converge.

Because the H-Matrix only needs a minimum 4 of points, it is an ideal selection for outlier rejection to reduce computational overhead. For example, we use the well-known RANSAC method with 4 randomly selected points in each iteration. Comparing with other commonly used methods, such as such as collinearity (CL) model which needs minimum 6 points, or F-Matrix which needs 8 points, the H-matrix can significantly reduce the number of iterations as shown in Table 1 for varying probabilities of good matches in the data set provided.

Table 1: RANSAC Convergence iterations required for a given Probability of good matches in data set, P(inlier)

P(inlier)	H-matrix	CL	F-Matrix
0.9	4	6	8
0.8	9	15	25
0.7	17	37	78
0.6	33	96	272
0.5	71	292	1177
0.4	178	1122	7025
0.3	566	6315	70188

Conclusions

We present a novel iterative algorithm using H-matrix pose estimation that compensates for non-planar 3D points. This new method has the following advantages:

1. It is simple and robust.
2. Its solution accuracy is equivalent to other well-known pose estimation solutions.
3. Because it needs minimum 4 points, it is an ideal solution for outlier rejection. Specifically, it is particularly useful for outlier rejection in TRN applications.

Acknowledgement. This research was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. © California Institute of Technology.

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